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without Resampling**

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# Zero-Intelligence Trading

## without Resampling

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**Abstract.** This paper studies the consequences of removing the resampling assumption from the zero-intelligence trading model in Gode and Sunder (1993). We obtain three results. First, individual rationality is no longer sufficient to attain allocative efficiency in a continuous double auction; hence, the *rules of the market* matter. Second, the allocative efficiency of the continuous double auction is higher than for other sequential protocols both with or without resampling. Third, compared to zero intelligence, the effect of learning on allocative efficiency is sharply positive without resampling and mildly negative with resampling.

**Keywords:** allocative efficiency, zero intelligence, market protocols.

**JEL Classification Numbers:** D40, D51, C70, C92.

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# 1 Introduction

In a recent paper, Mirowski (2007) argues that we are witnessing a “shift to a market-centered theory of computational economics” (p. 214). He attributes an important strand in this shift to the ramifications of Gode and Sunder (1993). This seminal paper is widely credited<sup>1</sup> with showing that the continuous double auction can attain allocative efficiency and convergence to the equilibrium price in the absence of trader intelligence. Such zero-intelligence (henceforth, ZI) is modeled by replacing human subjects with computerized agents that generate random quotes.

As Mirowski himself acknowledges, “there is still substantial dispute over the interpretation of their results” (p. 216); e.g., see Brewer et al. (2002). The boldest claim is that an appropriate market institution can override the cognitive limitations of individuals to achieve allocative efficiency and discover the equilibrium price. On the other side of the fence, the sharpest criticism is offered by Gjerstad and Shachat (2007). This paper provides a fresh and careful reading of Gode and Sunder (1993) that makes two points: first, convergence to the equilibrium price does not actually occur in Gode and Sunder (1993); second, the key condition for allocative efficiency is the *individual rationality* of the agents rather than the *market discipline* imposed by the continuous double auction.

Based on this, Gjerstad and Shachat (2007) concludes that “individual rationality is both necessary and sufficient to reach” allocative efficiency (p. 7). This argument is backed up by the claim that Gode and Sunder (1993) deals with a special case of the B-process for which Hurwicz, Radner and Reiter (1975) proves that in an economy without externalities a random but otherwise individually rational behavior converges to a Pareto optimal allocation.

In fact, this claim rests on a subtle but far from innocuous assumption made in Gode and Sunder (1993) that has gone largely unnoticed in the literature. We quote from Gode and Sunder (1993, p. 122): “There are several variations of the double auction. We made three choices to simplify our implementation of the double auction. Each bid, ask, and transaction was valid for a single unit. *A transaction canceled any unaccepted bids and offers.* Finally, when a bid and a ask crossed, the transaction price was equal to the earlier of the two.” (Emphasis added.) We call the emphasized assumption *resampling* because under zero intelligence it forces all agents who have already uttered a quote to issue a new (random) one after each transaction.

This paper studies the consequences of removing the resampling assumption. We obtain three results. First, under zero intelligence, individual rationality without resampling is not sufficient to attain allocative efficiency in

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<sup>1</sup> See Footnote 5 in Gjerstad and Shachat (2007).

a continuous double auction; hence, the *rules of the market* matter. On the other hand, with or without resampling, the allocative efficiency of the continuous double auction is higher than for the other sequential protocols we consider; hence, this market protocol is still the most effective among those. Third, when zero intelligence is replaced by a simple variant of the algorithm mimicking learning-based human behavior proposed in Gjerstad and Dickhaut (1998), we find that the effect on allocative efficiency is sharply positive without resampling but tends to be mildly negative with resampling.

## 2 The model

We use a setup inspired to Gode and Sunder (1993). There is an economy with a large number ( $n = 5000$ ) of traders, who can exchange single units of a generic good. Each agent is initialized to be a seller or a buyer with equal probability. Each seller  $i$  is endowed with one unit of the good for which he has a private cost  $c_i$  that is independently drawn from the uniform distribution on  $[0, 1]$ . Each buyer  $j$  holds no units and has a private valuation  $v_j$  for one unit of the good that is independently drawn from the uniform distribution on  $[0, 1]$ . By individual rationality, each seller  $i$  is willing to sell his unit at a price  $p \geq c_i$  and each buyer  $j$  is willing to buy at most one unit at a price  $p \leq v_j$ .

Gode and Sunder (1993) makes the three simplifying assumptions cited above. We maintain the first one and restrict all agents to trade at most one unit. The third assumption selects the continuous double auction as the market protocol that regulates the interactions between traders. We expand on this and compare the allocative efficiency of four different sequential protocols, including of course the continuous double auction. These four protocols are: the continuous double auction, a nondiscretionary dealership, a hybrid of these two, and the trading pit. The first three are described in detail in LiCalzi and Pellizzari (2006, 2007).

Briefly, in the continuous double auction (henceforth C) traders sequentially place quotes on the selling and buying books. Orders are immediately executed at the outstanding price if they are marketable; otherwise, they are recorded on the books with the usual price-time priority and remain valid until the end of the trading session. In the trading pit (henceforth T), traders are randomly matched in pairs: each agent in a pair utters a quote and, if compatible, they transact at a price equal to the average of their quotes; this transaction price is made known to the market, but its participants have no access to the offers exchanged within a pair.

In the dealership (henceforth, D) there is a specialist who posts bid and ask quotes valid only for a unit transaction. Agents arrive sequentially and can trade only at the dealer's quotes. Right after a transaction is completed, both dealer's quotes increase (or decrease) by a fixed amount  $k$  when the

agent completes a purchase (or a sale); hence, the bid-ask spread  $\Delta$  remains constant. Clearly, completing a trade between a buyer and a seller by going through the dealer is costly: for instance, if trader  $i$  sells one unit to the dealer that is immediately after resold to buyer  $j$ , the dealer pockets a value of  $\Delta - k$ . In this respect, the presence of the dealer negatively affects allocative efficiency. On the other hand, because the dealer guarantees a fixed bid-ask spread, it has a stabilizing effect on price dispersion that is usually beneficial.

For a large range of values, the force of these two effects vary in a predictable manner. Hence, the instantiation of  $k$  and  $\Delta$  is influential but not crucial: we assume  $k = 0.005$  and  $\Delta = 0.05$  throughout the paper. The choice of the initial dealer's quotes, instead, is more delicate: when these happen to be far away from the equilibrium price, the effect on allocative efficiency may be relevant because the first few trades tend to occur on the same side of the market (until the dealer's quotes get closer to the equilibrium price). Except for a final comment in Section 3.3, we mute this issue and assume that the initial quotes exactly straddle the (theoretical) equilibrium price. Finally, the hybrid market (henceforth, H) combines the continuous double auction with the dealership: agents have access to the dealer's quotes as well as to the offers from the public recorded in the book. The initialization for H is the same used for D; that is,  $k = 0.005$ ,  $\Delta = 0.05$  and the initial dealer's quotes straddle the equilibrium price.

Each of these four protocols is organized over a single trading session, where all agents participate. Their order of arrival is randomly selected. Whenever a transaction takes place between two agents, their own orders are removed from the market and the agents become inactive. The difference between assuming resampling or not is the following. Under no resampling, each agent gets only one chance to act: he can trade up to one unit (if he finds a suitable quote) or, limitedly to C and H, utter his own quote (that remains valid until the end). The market closes after all agents have had their chance to act. Under resampling as postulated in Gode and Sunder (1993), until an agent completes a trade and becomes inactive, the refresh following a trade may give him a new chance to act. Therefore, the number of chances for actions is much greater under resampling, and this tends to increase allocative efficiency. To minimize this bias, we assume that under resampling the market closes when, following a refresh, all the active agents have issued a quote and no transaction has occurred.

Two more differences separate the book-based (we call them "literate") protocols C and H from the "illiterate"<sup>2</sup> protocols D and T. First, the book in C and H offers to the current agent an option to store his quote, extending his opportunity to trade in the future; on the contrary, D and T limit his options to immediate trade or no trade at all. Second, the book makes quotes from

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<sup>2</sup> This terminology is non-standard, but less convoluted than a plain "non-book-based".

past traders available to the current agent, presenting him with a larger set of potential counterparts for his trade; on the other hand, for illiterate protocols the only available counterpart is the dealer in D and a single partner in T. In other words, a literate protocol expands the opportunities for trades as well as the pool of potential counterparts. These differences are not a crucial issue under resampling, because a trader returning to the market faces a new opportunity to trade, usually at different conditions. However, as we discuss below, they have a substantial effect when resampling is not allowed.

We use two different behavioral assumptions in our tests. Under zero intelligence, when an agent  $i$  must issue a quote, he picks a random number from the uniform distribution on  $[0, v_i]$  if he is a buyer and from the uniform distribution on  $[c_i, 1]$  if he is a seller. This behavior corresponds to zero intelligence under individual rationality and is called ZI-C in Gode and Sunder (1993). The second behavioral assumption is a simplified version<sup>3</sup> of the learning model introduced in Gjerstad and Dickhaut (1998), where each trader transforms the empirical acceptance frequencies to generate beliefs and then issue the quote that maximizes his expected surplus with respect to these beliefs. This approach is in general quite sensitive to fine details in its initialization and implementation. However, it can be calibrated to effectively mimic the basic features of human behavior in experimental trading markets. See Gjerstad (2007) for more details and an improved version of the original (1998) model.

Our implementation is the following. We discretize the unit interval  $[0, 1]$  for prices by assuming a “tick” equal to  $1/200 = 0.005$ . Let  $H_t(x) = \#(p \leq x)$  denote the empirical cumulative frequency of past transaction prices at time  $t$ . Each buyer  $i$  starts up with a uniform “prior” described by the cumulative distribution  $F_i(x) = \min\{(x - bv_i)^+, 1\}$  on the ticked prices contained in the interval  $[bv_i, v_i]$ , where  $b = 0.8$ . (For a seller  $i$ , we assume by symmetry a uniform distribution over the interval  $[c_i, (1 - b) + bc_i]$ .) This initial distribution is associated to a coefficient  $a_i$  that defines the stubbornness of  $i$ ’s initial beliefs; we assume that  $a_i$  is an integer drawn (once for each agent) from the uniform distribution on  $\{1, 2, \dots, 100\}$ . When a buyer  $i$  is called up for trading at time  $t$ , he combines his “prior” with the empirical distribution  $H_t(p)$  and derives a “posterior” cumulative distribution  $P(p \leq x)$  that is proportional to  $a_i F_i(x) + H_t(x)$ . Then buyer  $i$  issues a bid  $b$  that maximizes his expected utility  $(v - b) \cdot P(p \leq b)$ . (Sellers’ behavior is analogous.)

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<sup>3</sup> The most notable difference is that we do not assume bounded recall of past transactions.

### 3 Results

We are interested in the allocative efficiency of different market protocols under zero intelligence. As usual, allocative efficiency is defined as the ratio between the realized gains from the trade and the maximum feasible gains from trade. This measure is adimensional, facilitating comparisons. We compare the allocative efficiency of the four protocols described above under both zero intelligence and our version of the learning model proposed in Gjerstad and Dickhaut (1998). Since we view the role of the dealer as a mere feature of the protocol, his final gains/losses are not included in the computation of the allocative efficiency.

#### 3.1 Test 1: does resampling matter?

Assume zero-intelligence trading. The left-hand side of Figure 1 shows as datapoints the allocative efficiency of the continuous double auction *with resampling* for 100 different runs. The right-hand side shows the same information for the continuous double auction *without resampling*. The  $y$ -axes use the same scale, so that it is possible to directly compare the results under the two assumptions by visual inspection: the higher the level, the higher the allocative efficiency.

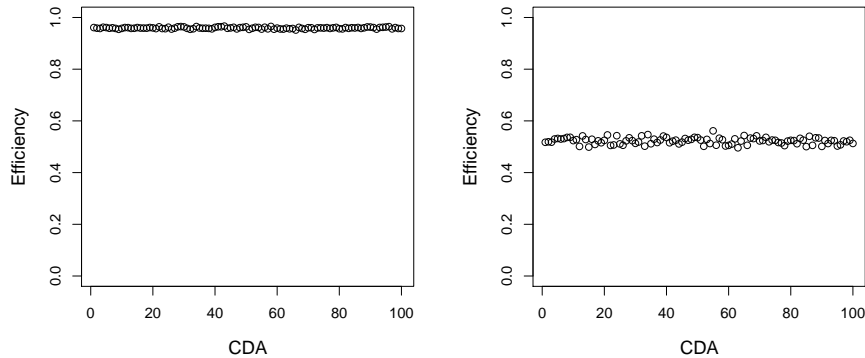


Figure 1: Allocative efficiency for C with (left) and without resampling (right).

The average allocative efficiency is 0.96 with resampling and 0.52 without resampling. Visual inspection strongly suggests that the distribution of the allocative efficiency with resampling stochastically dominates the distribution without resampling. More modestly, we claim that under resampling the expected value of allocative efficiency is higher. This is supported for any practical level of confidence by the directional version of the Wilcoxon signed-rank test. (Here and in the following, by a *practical* level of confidence we mean a  $p$ -value lower than  $10^{-5}$ .) Limited to our experiment,



therefore, we conclude that *ceteris paribus* resampling yields a higher allocative efficiency than no resampling. In short, resampling truly matters a lot.

### 3.2 Test 2: which protocol performs better under zero intelligence?

Our second test extends the first one by looking at the effects of resampling under zero intelligence for other sequential protocols. Each protocol is identified by its initials on the  $x$ -axis and by a different color: the continuous double auction (C) is in black; the nondiscretionary dealership (D) is in red; the hybridization (H) of the continuous double auction with a dealership is in green; and the trading pit (T) is in blue. The left-hand side of Figure 2 reports for each protocol the allocative efficiency *with resampling* for 100 different runs, as well as the sample average at the bottom of each column. The right-hand side shows the same information for the continuous double auction without resampling. Again, the  $y$ -axes use the same scale so direct comparison is possible.

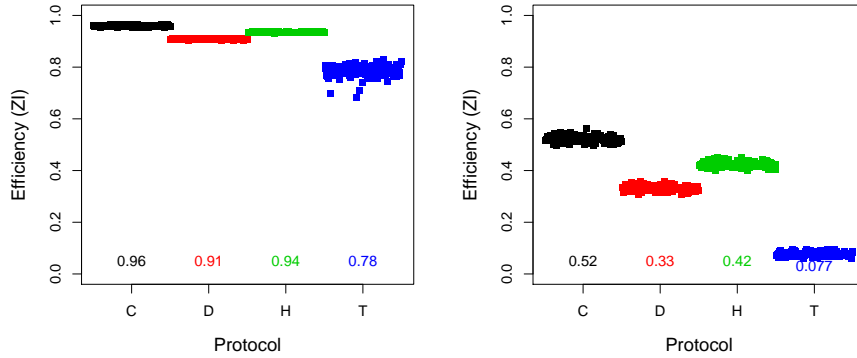


Figure 2: Allocative efficiency with (left) and without resampling (right).

We make two claims. The first one is that, for each protocol, allocative efficiency with resampling is significantly much higher than without resampling. This confirms and reinforces our earlier claim that the assumption of resampling matters a lot. The data in black concerning the continuous double auction (C) are reproduced from Figure 1 and need no further commentary. The data in red regarding the dealership (D) report a sample average of 0.91 with resampling against 0.33 with no resampling. Analogously, the data in green regarding the hybrid protocol (H) give a sample average of 0.94 with resampling against 0.42 with no resampling. Finally, the sample averages for the trading pit (T) is 0.78 with resampling and 0.077 with no resampling. For each protocol, the directional version of the Wilcoxon signed-rank test supports the significance of the difference with

and without resampling for any practical level of confidence.

We conclude that the introduction of the resampling assumption has a dramatic positive effect on allocative efficiency under zero intelligence. Hence, this assumption introduces an important bias that undermines Gode and Sunder’s (1998) claim that “the primary cause of the high allocative efficiency of double auctions is the market discipline imposed on traders” (p. 134), unless such market discipline is not taken to include resampling as well.

Two minor observations are worth making. First, regardless of the resampling assumptions, allocative efficiency is higher for literate protocols. The reason is that they give each trader access to a larger pool of counterparts. Second, the differences in the allocative efficiency of the trading pit are exaggerated by the minor modeling assumption that traders are matched in pairs. This implies that several pairs end up being formed by traders on the same side of the market who are bound not to trade. Therefore, we have also tested the alternative assumption that buyers and sellers are matched in pairs, making sure that each pair is formed by agents on the opposite side of the market. In this second case, the sample average of the allocative efficiency without resampling is 0.15. No qualitative conclusion is affected, although it is obvious that the trading pit works much better if traders can be screened in buyers versus sellers before being matched. (For each of the other protocols, the adirectional version of the Wilcoxon signed-rank test supports at any practical level of confidence the claim that it makes no difference for allocative efficiency to have buyers and sellers arrive in random order or alternately.)

Our second claim is that the allocative efficiency of the continuous double auction with or without resampling is higher than for other sequential protocols; hence, this market protocol remains more effective under zero intelligence. This is easily detectable by visual inspection of the two tables in Figure 2. The directional version of the Wilcoxon signed-rank test supports the claim that C yields a higher expected allocative efficiency than H (the highest competitor) for any practical level of confidence, both in the case of resampling (left) and no resampling (right). This confirms Gode and Sunder’s (1993) intuition that the continuous double auction provides an important and natural benchmark for allocative efficiency under zero intelligence. The next test inquires whether this remains true under more realistic assumptions about agents’ behavior.

### 3.3 Test 3: does learning make a difference?

Our third test extends the previous one by looking at the allocative efficiency of protocols under the alternative assumption that traders learn and optimize according to a slightly simplified version of the model in Gjerstad and Dickhaut (1998). We consider first the case without resampling, and

then the case with resampling.

The right-hand side of Figure 3 shows for each protocol the allocative efficiency without resampling for 100 different runs as well as the sample average, under the assumption that all agents base their trading on our simple model of learning and optimization. The left-hand side shows the same information (copied from Figure 2) under zero-intelligence trading for ease of comparison. The usual coding applies for initials and color.

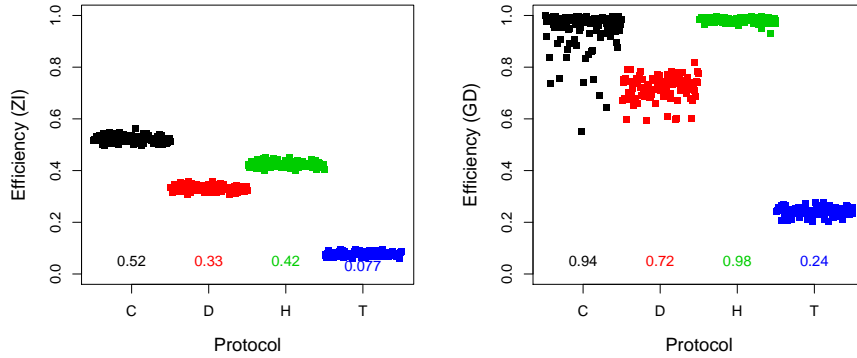


Figure 3: Allocative efficiency under heuristic learning without resampling.

Direct inspection shows that, without resampling, learning greatly improves the expected value of the allocative efficiency for each protocol. The sample average shoots from 0.52 to 0.94 for C, from 0.33 to 0.72 for D, from 0.42 to 0.98 for H, and from 0.077 to 0.24 for T. (The directional Wilcoxon test supports this hypothesis for any practical level of confidence.) This effect is easily explained. Under zero intelligence, resampling implies a complete refresh after each trade, in the sense that (conditional on the set of active traders) the probability distribution of the next quote is independent of the past. This makes the prices of the first few transactions almost irrelevant for predicting future behavior. On the other hand, under learning, the prices of past transactions affect the beliefs and hence the actions of future traders. The initial transaction prices feed future beliefs, amplifying the effect.

Even though the lack of resampling forbids agents from revising their past quotes, the learning process substitutes for this because incoming agents use past history when formulating their quotes. Therefore, under no resampling, learning makes a huge difference for the allocative efficiency of a market protocol. To the extent that learning is a behavioral assumption while no resampling is an institutional feature, this strongly suggests that we cannot apply the claim that “the primary cause of the high allocative efficiency of double auctions is the market discipline imposed on traders” (Gode and Sunder, 1993, p. 134) to situations in which resampling does not hold.

A comparison of the left- and right-hand sides of Figure 3 shows two more

effects. First, regardless of the behavioral assumptions, allocative efficiency is as usual higher for literate protocols. Second, learning-based behavior tends to increase the dispersion of allocative efficiency. The sample standard deviation goes from 0.013 to 0.081 for C, from 0.011 to 0.046 for D, from 0.011 to 0.010 for H, and from 0.008 to 0.016 for T. There is a sharp increase for three protocols and a mild decrease for one. This is another manifestation of the path-dependency implicit in our learning process: when a few initial trades off the equilibrium price point beliefs in the wrong direction, behavior may cluster around the wrong price and reduce allocative efficiency. This tends to increase the variability in performance, although the overall effect remains clearly favorable.

We now move to consider resampling. Figure 4 reports the same information as Figure 3 under the assumption of resampling. The left-hand side assumes zero intelligence; the right-hand side is based on our learning-based behavioral assumption. The usual coding applies. Once again, regardless of the behavioral assumptions, allocative efficiency is higher for literate protocols.

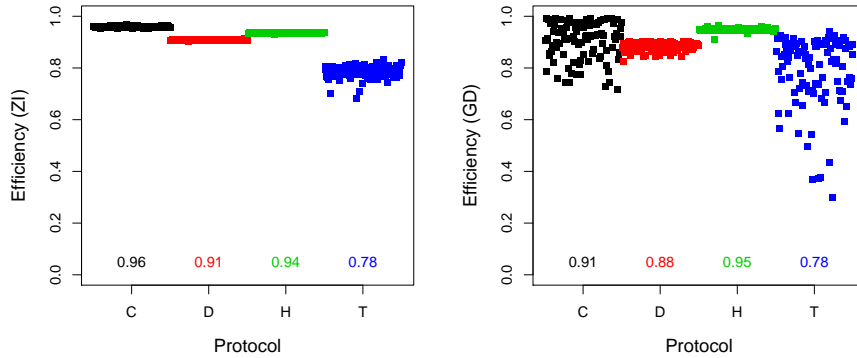


Figure 4: Allocative efficiency under heuristic learning with resampling.

Under resampling, learning degrades the expected allocative efficiency for three protocols. The sample average falls from 0.96 to 0.91 for C, from 0.91 to 0.88 for D, and from 0.7845 to 0.7838 for T, while it increases from 0.94 to 0.95 for H. (For C, D, and T, the directional Wilcoxon test supports the claim for any practical level of confidence; for H, the confidence level is 0.0007.) The sample standard deviation goes from 0.003 to 0.074 for C, from 0.002 to 0.017 for D, from 0.0019 to 0.006 for H, and from 0.024 to 0.139 for T.

Compared to the sharp improvement it carries without resampling, learning tends to have an opposite effect and bring about a reduction on allocative efficiency under resampling. Given the extremely high values of allocative efficiency under zero intelligence, this is not surprising. There is very little room to improve on allocative efficiency, so the rare occasions when path-

dependent beliefs fixate on the wrong price end up reducing the allocative efficiency and increasing its dispersion. This is however not true for the hybrid protocol, because it can exploit the traders' book to reduce the amount of allocative efficiency lost to the dealer as well as the presence of the dealer herself to reduce the chance of transaction prices fixating on the wrong price.

The effects of path-dependent learning can also be appraised by comparing the time series of the transaction prices. The left-hand (respectively, right-hand) side of Figure 5 shows two representative series for the continuous double auction with (without) resampling: zero intelligence is in red, learning-based behavior is in black. The  $x$ -axis reports the numbers of transactions: the longer the series, the higher the volume.

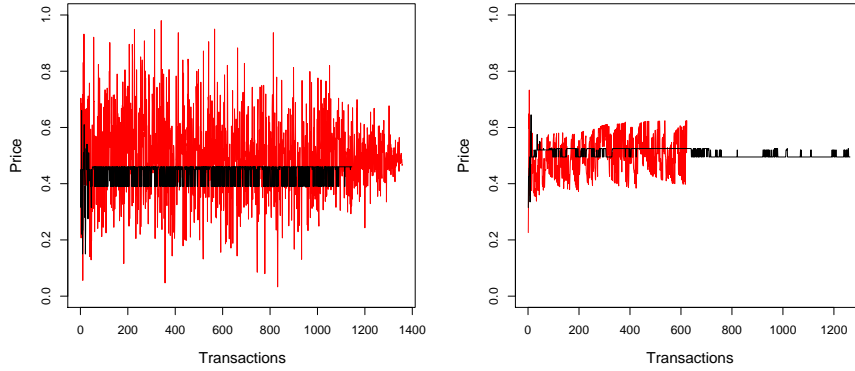


Figure 5: Series of transaction prices for C with (left) and without resampling (right).

Zero intelligence exhibits comparatively wilder oscillations that eventually tend to fade out, but are centered around the correct equilibrium price. Moreover, as obvious, the volume generated with resampling is much higher than without resampling. Finally, it is apparent that lack of resampling sharply reduces the overall dispersion of transaction prices. A careful look at the right-end tail of the series on the left shows that the dispersion with resampling becomes comparable to that one without resampling precisely when trading in the latter approaches its end.

Learning-based behavior generates much tighter series around some price. Allocative efficiency is hurt in those relatively unfrequent cases where this is different from (on the left of Figure 5, lower than) the equilibrium price. Independently of the assumptions about resampling, whenever beliefs fixate around the “wrong” price, the volume of transactions goes down and this hurts allocative efficiency. But there are two crucial differences. First, without resampling, traders cannot re-enter the market after having issued a quote: as wrong beliefs affect less people, the effect is reduced. Second, and limitedly to the two illiterate protocols (D and T), volume without re-

	$p = 0.5$				$p = 0.03$			
	zero intelligence		learning		zero intelligence		learning	
	D	H	D	H	D	H	D	H
no resampling	0.33	0.42	0.72	0.98	0.37	0.49	0.10	0.23
with resampling	0.91	0.94	0.88	0.95	0.94	0.97	0.13	0.26

Table 1: The impact of the dealer’s initial quotes on allocative efficiency.

sampling is heavily impaired by the combination of potentially wrong beliefs with uniqueness of the opportunity to trade and of the potential counterpart. As a result, given learning based-behavior, giving up resampling has a moderate positive effect for allocative efficiency in the two literate protocols C and H and a large negative effect for the illiterate protocols D and T.

A related issue is the robustness of our learning-based model for protocols involving a dealer in extreme situations, such as when the initial dealer’s quote are very far from the equilibrium price. For instance, Table 1 summarizes the sample averages of allocative efficiency for D and H with and without resampling as well as under zero intelligence or learning-based behavior under two different initializations. On the left we report the case where the initial dealer’s quotes straddle the equilibrium price  $p = 0.5$ , as assumed throughout this paper. On the right, we report the case where the initial dealer’s quotes straddle  $p = 0.03$ . It is clear that the quantitative effects of extreme initializations for D and H may be substantial.

To sum it up, compared to zero-intelligence, learning-based behavior brings about two effects on the allocative efficiency of a protocol. The negative one is the occasional clustering of path-dependent beliefs around the wrong price. This reduces allocative efficiency. The positive effect is that beliefs fixating around the equilibrium price help future traders to avoid wrong quotes. This improves allocative efficiency. Without resampling, the positive effect swamps the negative one for all protocols. Under resampling, the negative effect tends to prevail. (An additional effect related to the literacy of a protocol emerges when comparing allocative efficiency under learning-based behavior with and without resampling.) While this is not the place for sweeping generalizations, it seems legitimate to conjecture that the performance of a protocol under zero intelligence *with* resampling is not a good proxy for its performance under learning-based human behavior *without* resampling.

## 4 Conclusions

Our first result is that, under zero intelligence, individual rationality without resampling is not sufficient to attain allocative efficiency in the continuous double auction. Stronger than that, it is not sufficient in none of the four market protocols we study: none of the simulations reported on the left

achieves more than 60% of the maximum allocative efficiency. This establishes that the *rules of the market* matter: when resampling is ruled out, zero intelligence rules out allocative efficiency. The “second” assumption in Gode and Sunder (1993) is not a mere simplification, but an important restriction.

Moreover, as shown in Section 3.2, assuming resampling on top of zero intelligence leads to a sharp increase in allocative efficiency for any of the four different protocols we tested. Therefore, the assumption of no resampling introduces a substantial bias towards achieving allocative efficiency. Gode and Sunder’s (1998) claim that “the primary cause of the high allocative efficiency of double auctions is the market discipline imposed on traders” (p. 134) does not hold when their unobtrusive assumption of resampling is dropped.

On the other hand, we also find that the allocative efficiency of the continuous double auction under zero intelligence is never lower than for the other three market protocols. This validates Gode and Sunder’s (1993) intuition about the effectiveness of the continuous double auction. In some cases, there may be other market protocols that exhibit similar results in this respect. This suggests the importance of introducing additional performance measures to rank market protocols that exhibit a similar degree of allocative efficiency, as discussed in LiCalzi and Pellizzari (2007a).

Our final result concerns the performance of the four market protocols when zero intelligence is replaced by a simple version of the learning-based model of human behavior proposed in Gjerstad and Dickhaut (1998). We find that learning-based behavior has two opposite effects on allocative efficiency. When path-dependent beliefs end up clustering around the wrong price, allocative efficiency is reduced. When instead they fixate around the equilibrium price allocative efficiency improves. The sign for the combination of these two effects is *a priori* ambiguous. We find that it is sharply positive without resampling and (usually) mildly negative with resampling. Since human behavior is very likely to have more in common with learning than with zero intelligence, this implies that the ability of a protocol to steer human traders towards allocative efficiency is not independent of the assumptions about resampling.

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